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Radiative entropy production—lost heat into entropy

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Abstract—Heat flow δQ of the First Law of Thermodynamics is expressed in terms of the entropy flow $\delta(Q/T)$

$$\delta Q \equiv \delta[T(Q/T)] = T\delta(Q/T)+(Q/T)\,\mathrm{d}T$$

where $T\delta(Q/T)$ denotes the energy equivalent of the entropy flow, and $(Q/T)\,\mathrm{d}T$ introduces the concept of lost heat into entropy production. Here $Q = Q^{\mathrm{K}}+Q^{\mathrm{R}}$ where superscripts K and R indicate conduction and radiation, respectively. In terms of the lost heat, dimensionless entropy productions on the wall of a thermal boundary layer and in a quenched laminar flame are respectively shown to be

$$\Pi_x \sim (1+q_x^{\mathrm{R}}/q_x^{\mathrm{K}})Nu_x^2 \quad \text{and} \quad \Pi_s \sim (1+q^{\mathrm{R}}/q^{\mathrm{K}})\,Pe^{-2}$$

where q^{R} and q^{K} are the one-dimensional fluxes associated with Q^{R} and Q^{K}, Nu_x is a local Nusselt number, and Pe is a Peclet number based on the laminar flame speed at the adiabatic flame temperature. The *tangency condition*, $\partial Pe/\partial T_{\mathrm{b}} = 0$, customarily used in the evaluation of minimum quench distance without any physical justification, is shown to correspond to *an extremum in entropy production.*

1. INTRODUCTION

THE FOUNDATIONS of entropy production go back to Clausius and Kelvin's studies on the irreversible aspects of the Second Law of Thermodynamics. Separately, the foundations of gas radiation date back to Rayleigh's studies on the illumination and polarization of the sunlit sky. Since then the theories based on these foundations have rapidly grown first by the efforts of natural philosophers followed by astrophysicists, and later by those of applied scientists and engineers. However, the entropy production associated with gas radiation apparently remained untreated and is the motivation of this study. Here the difference between the enclosure radiation (which neglects media participation) and the gas radiation which involves some optical thickness (or volumetric absorption) should be noted. This study deals only with the entropy production associated with gas radiation.

As is well known, the entropy production results from dissipative processes (involving mass, species, momentum and/or heat transfer, electromagnetic or nuclear transport). Less known is the fact that the dissipation may have a diffusive or hysteretic origin, the diffusion being directional and the hysteresis being cyclic. However, except for a few cases (such as strain hardening and the magnetic saturation), the majority of dissipative processes, including the dissipation of radiation, is of diffusive nature. A recent study by Arpaci [1] shows, in terms of the radiative stress obtained from the specular (kinetic) moments of the transfer equation, the diffusive nature of radiation for any optical thickness. Accordingly, the expression to be developed for entropy production is in terms of this stress, and includes also the dissipation resulting from conduction of heat and other diffusion processes.

The study consists of eight sections: following this introduction, Section 2 explains the thermodynamic foundations of the entropy production, Section 3 deals with a brief review of the radiative stress, Section 4 develops the transport aspects of entropy production in terms of this stress, Section 5 introduces some dimensionless numbers for radiation, Section 6 applies the entropy production to radiative heat transfer, Section 7 employs an extremum in entropy production for the interpretation of the *tangency condition* of laminar flame quenching, and Section 8 concludes the study.

2. THERMODYNAMIC FOUNDATIONS

There is a renewed interest in the Second Law of Thermodynamics, especially in its application to engineering problems. Because of its size, no attempt is made here for a review of the literature. However Bejan's extensive work [2, 3] on the interpretation of a variety of heat and fluid flow problems in terms of entropy production deserves special recognition. An inspection of this literature reveals that the concept of *lost heat* as opposed to that of *lost work* appears to remain untreated except for the recent presentations by Arpaci [4, 5] and Arpaci and Selamet [6]. The purpose of this section is to introduce the concept of lost heat, show the relation between this concept and the entropy production, and include the effect of radiation to this production.

Under the influence of thermal effects only, the First Law for a system with fixed boundaries gives

$$\mathrm{d}U = \delta Q + \mathrm{d}U_{\mathrm{g}} \tag{1}$$

NOMENCLATURE

B	Boltzmann number
B	equilibrium intensity, $4E_b$
c	velocity of light
c_p	specific heat at constant pressure
d	thickness of reaction zone
E_b	black body emissive power
E_n	integro-exponential function of order $n = 2, 3, 4$
f_i	body force
H	heat transfer number
i	complex unit
I	intensity
J	averaged intensity
k	thermal conductivity
k_0	wave number
k_i	wave number in x_i
l	characteristic length
l_i	unit vector in x_i
$M_{ijpq...}$	operator defined by equation (19)
Nu	Nusselt number
p	pressure
P	Planck number
Pe	Peclet number
q_i	heat flux in x_i
Q	heat
s	entropy/mass
s'''	rate of entropy generation/volume
s_{ij}	rate of deformation
S	entropy
S^0	laminar flame speed at adiabatic flame temperature
t	time
T	temperature
u	internal energy/mass or volume
u'''	rate of energy generation/volume
U	internal energy
v	specific volume
v_i	velocity in x_i
W	work
x_i	coordinate axis.

Greek symbols

α	thermal diffusivity
δ	thickness of boundary layer
Δ	quench distance
ε	emissivity
η	weighted nongrayness, $(\kappa_P/\kappa_R)^{1/2}$
κ	absorption coefficient
Π_{ij}	radiative tensor
Π	entropy number
ρ	density or reflectivity
σ	Stefan–Boltzmann constant
τ	optical thickness
τ_{ij}	stress
Ω	solid angle.

Subscripts

b	burned
g	generation
M	mean
P	Planck mean
R	Rosseland mean
s	entropy
u	unburned
w	wall
x	local
∞	ambient.

Superscripts

C	convection
K	conduction
R	radiation.

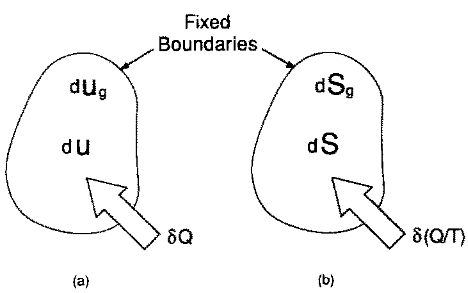

Fig. 1. First and Second Laws of Thermodynamics for a system of constant volume.

where dU_g denotes implicitly the energy generation resulting from dissipation of all non-thermal (mechanical, chemical, electromagnetic and nuclear) forms of energy into heat (Fig. 1(a)). For the same system, the Second Law is

$$dS = \delta(Q/T) + dS_g \tag{2}$$

dS_g being the entropy production (Fig. 1(b)). The other notation in equations (1) and (2) is conventional. Recognizing that the heat flow as a useful practical concept but the entropy flows as a fundamental concept, express the former in terms of the latter by considering the following identity

$$\delta Q \equiv \delta[T(Q/T)] = T\delta(Q/T) + (Q/T)\,dT. \tag{3}$$

Then, the First Law may be rearranged in terms of the entropy flow as

$$dU = T\delta(Q/T) + [(Q/T)\,dT + dU_g] \tag{4}$$

where, the second term on the right can be interpreted as the dissipation of heat into entropy (Fig. 2(a)). Hereafter, the dissipated heat will be called the *lost*

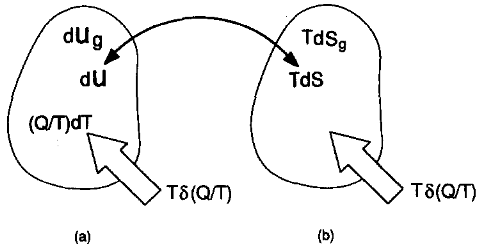

FIG. 2. (a) The First Law in terms of entropy flow and lost heat. (b) The energy equivalent of the Second Law.

heat. Also, for convenience to follow, write the energy equivalent of the Second Law (equation (2) multiplied by T (Fig. 2(b))) as

$$T\mathrm{d}S = T\delta(Q/T)+T\mathrm{d}S_g. \quad (5)$$

Now, consider the fundamental difference

$$\text{First Law}-(\text{Second Law})T \quad (6)$$

which gives

$$\text{Energy dissipation} \equiv (\text{Entropy production})T \quad (7)$$

and, as a special case

$$\text{Thermal energy dissipation} \equiv \text{Lost heat} \equiv (\text{Thermal entropy production})T. \quad (8)$$

Inserting equations (4) and (5) into equation (6) yields

$$\mathrm{d}U-T\mathrm{d}S = (Q/T)\,\mathrm{d}T+\mathrm{d}U_g-T\mathrm{d}S_g. \quad (9)$$

For a reversible process, all forms of dissipation vanish, and equation (9) is reduced to

$$\mathrm{d}U-T\mathrm{d}S = 0 \quad (10)$$

which is the Gibbs thermodynamic relation. For an irreversible process, this relation *among thermodynamic properties* continues to hold, and equation (9) gives the entropy production

$$\mathrm{d}S_g = \frac{1}{T}[(Q/T)\,\mathrm{d}T+\mathrm{d}U_g] \quad (11)$$

the first term in brackets being the lost heat to entropy production. Clearly, the entropy production has two parts, one resulting from the dissipation of all non-thermal (mechanical, chemical, electromagnetic and nuclear) forms of energy into heat and the other from the dissipation of heat into entropy.

Now, consider the radiation to be an ideal gas interacting with matter (gas). Let the internal energy, heat and work associated with radiation gas be U^R, Q^R and W^R, respectively. It can be shown by the consideration of the explicit relations between (U^R, Q^R, W^R) and the photon intensity that

$$U^R \ll U, \quad Q^R \sim Q^K, \quad W^R \ll W$$

provided the characteristic transport velocity remains much less than the velocity of light. Accordingly, under the influence of radiation

$$Q = Q^K+Q^R \quad (12)$$

Q^K being the heat flow by conduction. The next section is devoted to a brief review on the radiative stress and the description of the radiative heat transfer in terms of this stress.

3. RADIATIVE STRESS

The following brief review is in terms of spectrally averaged radiative properties and applies to continuous radiation. In view of the basic nature of the present study, the monochromatic aspects of radiation which are needed for practical cases involving approximate line (or band) models are not taken into account (see, for example, Tien and Lee [7] for an extensive review on these models).

The spectrally averaged definitions of the radiative internal energy, heat flux and stress in terms of the intensity I are

$$u^R = \frac{1}{c}\int_\Omega I\,\mathrm{d}\Omega = \frac{1}{c}J \quad (13)$$

$$q_i^R = \int_\Omega I l_i\,\mathrm{d}\Omega \quad (14)$$

$$\tau_{ij}^R = \frac{1}{c}\int_\Omega I l_i l_j\,\mathrm{d}\Omega = \frac{1}{c}\Pi_{ij} \quad (15)$$

where the J-scalar and the Π_{ij}-tensor are introduced for notational convenience, c is the velocity of light, and Ω is the solid angle. In terms of these definitions, the first three specular moments of the transfer equation are

$$\frac{\partial q_i^R}{\partial x_i} = \kappa_P(B-J) \quad (16)$$

$$\frac{\partial \Pi_{ij}}{\partial x_j} = -\kappa_R q_i^R \quad (17)$$

$$\Pi_{ij} = \frac{1}{3}B\delta_{ij}+\sum_{n=1}^{\infty}\frac{1}{\kappa_M^{2n}}\left(M_{ijpq\ldots}\frac{\partial}{\partial x_p}\frac{\partial}{\partial x_q}\right)B \quad (18)$$

with

$$M_{ijpq\ldots} = \frac{1}{4\pi}\int_\Omega (l_i l_j l_p l_q\ldots)\,\mathrm{d}\Omega. \quad (19)$$

Here $B=4E_b$, $E_b=\sigma T^4$ being the Stefan–Boltzmann law for the black body emissive power, κ_P and κ_R are the Planck and Rosseland means of the absorption coefficient, respectively, and $\kappa_M = (\kappa_P\kappa_R)^{1/2}$ is the geometric mean of these coefficients. The incorporation of κ_P and κ_R into the foregoing equations is discussed by Traugott [8], Cogley *et al.* [9], and their use in a variety of problems by Arpaci and co-workers [10–15]. Clearly, equation (16) denotes the thermal balance, equation (17) the momentum balance associated with radiation, and equation (18) gives the Π_{ij}-tensor in terms of a series based on specular moments.

Note that the radiative heat flux given by equation (17), rearranged as

$$q_i^R = -\frac{1}{\kappa_R}\frac{\partial \Pi_{ij}}{\partial x_j} \tag{20}$$

can be interpreted as a generalized diffusion process for any optical thickness. A procedure for the evaluation of equation (19) in terms of the Wallis integrals is described in Unno and Spiegel [16]. After lengthy manipulations, this procedure leads to

$$\Pi_{ij} = \sum_{n=0}^{\infty} \frac{\nabla^{2n-2}(2n\partial_i\partial_j + \nabla^2\partial_{ij})B}{\kappa_M^{2n}(2n+1)(2n+3)} \tag{21}$$

where $\partial_i = \partial/\partial x_i$ and $\partial_j = \partial/\partial x_j$ are used for notational convenience. The same result may be found also in earlier works (see, for example, Milne [17]). The formal similarity of equation (21) to the Hookean constitution for elastic solids should be noted.

An alternate form for this stress may be given in terms of the isotropic radiative pressure. First, invoking the assumption of isotropy, equations (13) and (15) are related as

$$\tau_{ij}^R = \tfrac{1}{3}u^R\delta_{ij} \tag{22}$$

which implies

$$\Pi_{kk} = J \tag{23}$$

where

$$\frac{1}{3c}\Pi_{kk} = -p \tag{24}$$

is the (isotropic) pressure of radiation. Then, from the trace of Π_{ij}, noting that $l_k l_k = 1$

$$\Pi_{kk} = \sum_{n=0}^{\infty}\left(\frac{\nabla^2}{\kappa_M^2}\right)^n \frac{B}{(2n+1)}. \tag{25}$$

Now, in a manner similar to the inclusion of the isotropic pressure to the development of viscous stress from elastic stress (see, for example, Arpaci and Larsen [18]), adding the identity

$$\tfrac{1}{3}J\delta_{ij} - \tfrac{1}{3}\Pi_{kk}\delta_{ij} = 0 \tag{26}$$

to equation (21), the Π_{ij}-tensor may be rearranged in terms of the radiation pressure

$$\Pi_{ij} = \tfrac{1}{3}J\delta_{ij} + \sum_{n=0}^{\infty}\frac{2n\nabla^{2n-2}(\partial_i\partial_j - \frac{1}{3}\nabla^2\delta_{ij})B}{\kappa_M^{2n}(2n+1)(2n+3)}. \tag{27}$$

The formal similarity of equation (27) to the viscous (Stokesean) stress and the electromagnetic (Maxwell) stress should be noted. This similarity is to be expected in view of the assumed isotropy for the elastic, viscous and electromagnetic continua (see, for example, Stratton [19] and Prager [20]). The use of the first term of equation (27) in place of equation (21) is the well-known Eddington approximation which leads to a diffusive heat flux

$$q_i^R = -\frac{1}{3\kappa_R}\frac{\partial J}{\partial x_i} \tag{28}$$

for any optical thickness. The maximum deviation of this flux from the exact flux given by equation (20) is about 29% at $\tau = 1/\sqrt{3}$ (see Arpaci [4]). The next section develops an expression for the radiative entropy production in terms of Π_{ij} given by equations (21) and (28).

4. LOCAL ENTROPY PRODUCTION

The entropy production discussed in Section 2 is extended here to moving media which requires as well the consideration of the *momentum balance*. For the Stokesean fluid, this balance in terms of the usual nomenclature is

$$\rho\frac{Dv_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i. \tag{29}$$

The *entropy balance* (the Second Law balanced by the local entropy production) is

$$\rho\frac{Ds}{Dt} = -\frac{\partial}{\partial x_i}\left(\frac{q_i}{T}\right) + s''' \tag{30}$$

where s''' denotes the local entropy production. Also, the *conservation of total* (thermomechanical) *energy* (or the First Law) including the heat flux expressed in terms of the entropy flux

$$\frac{\partial q_i}{\partial x_i} \equiv \frac{\partial}{\partial x_i}\left[\left(\frac{q_i}{T}\right)T\right] = T\frac{\partial}{\partial x_i}\left(\frac{q_i}{T}\right) + \left(\frac{q_i}{T}\right)\frac{\partial T}{\partial x_i} \tag{31}$$

is

$$\rho\frac{D}{Dt}(u + \tfrac{1}{2}v_i^2) = -\frac{\partial}{\partial x_i}\left[\left(\frac{q_i}{T}\right)T\right] - \frac{\partial}{\partial x_i}(pv_i) + \frac{\partial}{\partial x_j}(\tau_{ij}v_i) + \rho f_i v_i + u'''. \tag{32}$$

Now, the fundamental difference

$$\text{Total energy} - (\text{Momentum})v_i - (\text{Entropy})T \tag{33}$$

in terms of equations (29), (30), (32) and the *conservation of mass*

$$\frac{D\rho}{Dt} + \rho\frac{\partial v_i}{\partial x_i} = 0 \tag{34}$$

yields

$$\rho\left(\frac{Du}{Dt} - T\frac{Ds}{Dt} + p\frac{Dv}{Dt}\right) = -\left(\frac{q_i}{T}\right)\frac{\partial T}{\partial x_i} + \tau_{ij}s_{ij} + u''' - Ts''' \tag{35}$$

where s_{ij} is the rate of deformation. For a reversible process, all forms of dissipation vanish, and

$$\left(\frac{Du}{Dt} - T\frac{Ds}{Dt} + p\frac{Dv}{Dt}\right) = 0 \tag{36}$$

which is the Gibbs thermodynamic relation. For an irreversible process, equation (36) continues to hold provided the process can be assumed in *local equilibrium*. Then, the local entropy production is found to be

$$s''' = \frac{1}{T}\left[-\left(\frac{q_i}{T}\right)\left(\frac{\partial T}{\partial x_i}\right)+\tau_{ij}s_{ij}+u'''\right] \quad (37)$$

where the first term in brackets denotes the dissipation of thermal energy into entropy (lost heat), the second term denotes the dissipation of mechanical energy into heat (lost work), and the third term denotes the dissipation of any (except thermomechanical) energy into heat. When radiation is appreciable, q_i denotes the total flux involving the sum of the conductive flux and the radiative flux

$$q_i = q_i^{\mathrm{K}} + q_i^{\mathrm{R}}. \quad (38)$$

In terms of the usual conductive constitution and the radiative constitution given by equation (20), the local entropy production is found to be

$$s''' = \frac{1}{T}\left[\frac{1}{T}\left(k\frac{\partial T}{\partial x_i}+\frac{1}{\kappa_{\mathrm{R}}}\frac{\partial \Pi_{ij}}{\partial x_j}\right)\left(\frac{\partial T}{\partial x_i}\right)+\tau_{ij}s_{ij}+u'''\right] \quad (39)$$

the radiative part of which needs to be related to temperature through equation (21) or (27). Also, the considerations of only the first term of equation (27) yields

$$s''' = \frac{1}{T}\left[\frac{1}{T}\left(k\frac{\partial T}{\partial x_i}+\frac{1}{3\kappa_{\mathrm{R}}}\frac{\partial J}{\partial x_i}\right)\left(\frac{\partial T}{\partial x_i}\right)+\tau_{ij}s_{ij}+u'''\right] \quad (40)$$

the radiative part of which is Eddington approximated and needs to be coupled with

$$(\nabla^2 - 3\kappa_{\mathrm{M}}^2)J = -12\kappa_{\mathrm{M}}^2 E_{\mathrm{b}} \quad (41)$$

(see, for example, Arpaci and Gözüm [11]).

5. QUALITATIVE RADIATION

This section is devoted to some qualitative arguments which will prove useful in the following two sections. Reconsider only the thermal part of equation (40)

$$s''' = \frac{1}{T}\left[\frac{1}{T}\left(k\frac{\partial T}{\partial x_i}+\frac{1}{3\kappa_{\mathrm{R}}}\frac{\partial j}{\partial x_i}\right)\left(\frac{\partial T}{\partial x_i}\right)\right]. \quad (42)$$

Introduce an entropy production number

$$\Pi_s = \frac{s''' l^2}{k} \quad (43)$$

l being a characteristic length, and a heat transfer number

$$H = \frac{(\partial J/\partial x_i)/3\kappa_{\mathrm{R}}}{k(\partial T/\partial x_i)} = \frac{q_i^{\mathrm{R}}}{q_i^{\mathrm{K}}}. \quad (44)$$

In terms of these numbers, equation (42) becomes

$$\Pi_s = (1+H)\frac{l^2}{T^2}\left(\frac{\partial T}{\partial x_i}\right)\left(\frac{\partial T}{\partial x_i}\right). \quad (45)$$

To proceed further, a dimensional interpretation of q_i^{R} is needed. From equation (28)

$$q^{\mathrm{R}} \sim \frac{J_{\mathrm{w}} - J_\infty}{3\kappa_{\mathrm{R}}\delta} \quad (46)$$

where δ is the thickness of thermal boundary layer, J_{w} and J_∞ are the wall and ambient values of J, respectively. To relate J to temperature, consider the radiative constitution given by equation (41). By the help of Fourier transforms, for example

$$\exp(\mathrm{i}k_j x_j)$$

$\mathrm{i} = \sqrt{-1}$ and k_j being the wave number vector

$$\nabla^2 = -k_0^2, \quad k_0^2 = k_1^2 + k_2^2 + k_3^2$$

or, in view of $k_0 \sim \delta^{-1}$

$$\nabla^2 \sim -\delta^{-2}$$

and equation (41) yields

$$(\delta^{-2} + 3\kappa_{\mathrm{M}}^2)J \sim 12\kappa_{\mathrm{M}}^2 E_{\mathrm{b}} \quad (47)$$

Then, in terms of the optical thickness

$$\tau \sim k_{\mathrm{M}}\delta \quad (48)$$

$$J \sim \left(\frac{12\tau^2}{1+3\tau^2}\right)E_{\mathrm{b}} \quad (49)$$

which, together with equation (46), leads to the radiative heat flux

$$q^{\mathrm{R}} \sim 4\eta\left(\frac{\tau}{1+3\tau^2}\right)(E_{\mathrm{bw}} - E_{\mathrm{b}\infty}) \quad (50)$$

valid for any optical thickness. However, this relation does not include any boundary effect.

To include this effect into equation (50), first consider the boundary affected thick gas and thin gas approximations. For the thick gas, from Arpaci [21] and Arpaci and Larsen [22]

$$q_y^{\mathrm{R}} = -\frac{4}{3\kappa_{\mathrm{R}}}(1 - \tfrac{1}{2}\rho_{\mathrm{w}}E_3 - \tfrac{3}{2}E_4)\frac{\partial E_{\mathrm{b}}}{\partial y} \quad (51)$$

where ρ_{w} is the wall reflectivity, E_3 and E_4 are the usual exponential integrals of order three and four. On boundaries

$$q_y^{\mathrm{R}}|_{\mathrm{w}} = -\frac{4}{3\kappa_{\mathrm{R}}}\left(\frac{\varepsilon_{\mathrm{w}}}{2}\right)\frac{\partial E_{\mathrm{b}}}{\partial y}\bigg|_{\mathrm{w}} \quad (52)$$

or, dimensionally

$$q_{\mathrm{w}}^{\mathrm{R}} \sim \frac{4\eta}{3\tau}\left(\frac{\varepsilon_{\mathrm{w}}}{2}\right)(E_{\mathrm{bw}} - E_{\mathrm{b}\infty}) \quad (53)$$

where $\eta = (\kappa_P/\kappa_R)^{1/2}$. For the thin gas, from Lord and Arpaci [10]

$$\frac{\partial q_y^R}{\partial y} = 4\kappa_P\left[(E_b - E_{b\infty}) - \frac{\varepsilon_w}{2}(E_{bw} - E_{b\infty})E_2\right] \quad (54)$$

where E_2 is the exponential integral of order two. Outside of a thermal boundary layer, $E_b \sim E_{b\infty}$, and equation (54) is reduced to

$$\frac{\partial q_y^R}{\partial y} = -4\kappa_P\left(\frac{\varepsilon_w}{2}\right)(E_{bw} - E_{b\infty})E_2 \quad (55)$$

or, near boundaries

$$\left.\frac{\partial q_y^R}{\partial y}\right|_w = -4\kappa_P\left(\frac{\varepsilon_w}{2}\right)(E_{bw} - E_{b\infty}) \quad (56)$$

which, on dimensional grounds, yields

$$q_w^R \sim 4\eta\tau\left(\frac{\varepsilon_w}{2}\right)(E_{bw} - E_{b\infty}). \quad (57)$$

The comparison of equations (53) and (57) with the thick gas and thin gas limits of equation (50) identifies the boundary effect by the emissivity factor $\varepsilon_w/2$. Accordingly, the radiative heat flux including the wall as well as the emission and absorption effects is found to be

$$q^R \sim 4\eta\left(\frac{\varepsilon_w}{2}\right)\left(\frac{\tau}{1+3\tau^2}\right)(E_{bw} - E_{b\infty}). \quad (58)$$

Furthermore, introducing the Planck number

$$P_w = \frac{\text{Emission}}{\text{Conduction}} \sim \frac{E_{bw} - E_{b\infty}}{k(T_w - T_\infty)/\delta} \quad (59)$$

equation (44) may be rearranged as

$$H_w = \frac{q_w^R}{q_w^K} \sim 4\eta\left(\frac{\varepsilon_w}{2}\right)\left(\frac{\tau}{1+3\tau^2}\right)P_w. \quad (60)$$

Finally, equation (45) yields in terms of equation (60)

$$\Pi_s \sim \left(\frac{T_w - T_\infty}{T}\right)^2(1 + H_w) \quad (61)$$

or, explicitly

$$\Pi_s \sim \left(\frac{T_w - T_\infty}{T}\right)^2\left[1 + 4\eta\left(\frac{\varepsilon_w}{2}\right)\left(\frac{\tau}{1+3\tau^2}\right)P_w\right]. \quad (62)$$

The smallest value of this production is on the hot boundary, and its radiative part becomes, after some rearrangement

$$\frac{\Pi_s}{2\eta\varepsilon_w P_w} \sim \left(\frac{T_w - T_\infty}{T_w}\right)^2\left(\frac{\tau}{1+3\tau^2}\right). \quad (63)$$

For a proportionality constant of unity (chosen arbitrarily for a graphical representation of equation (63)), Fig. 3 shows the boundary production of radiative entropy vs the optical thickness and the temperature ratio. The foregoing dimensional considerations will prove useful in the next two sections on the evaluation of entropy production near a boundary, and across a discontinuity (such as flames).

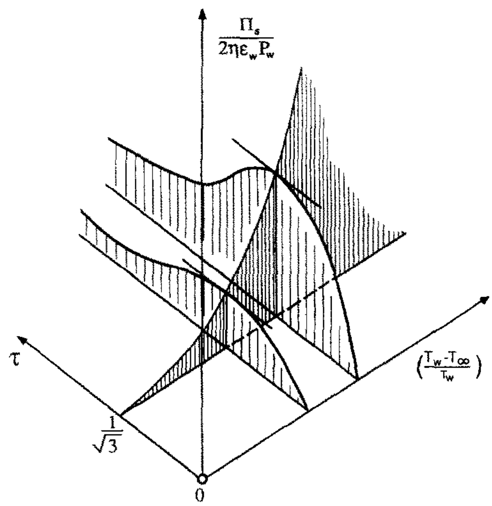

FIG. 3. Radiative entropy production.

6. HEAT TRANSFER

Consider a thermal boundary layer of local thickness δ next to a wall (Fig. 4). On dimensional grounds, the local thermal entropy production (recall equations (37) and (38)) on the wall is

$$s_x''' \sim \frac{1}{T_w}\left(\frac{q_w^K + q_w^R}{T_w}\right)\left(\frac{T_w - T_\infty}{\delta}\right) \quad (64)$$

subscripts w and ∞ indicating wall and ambient. Rearrange equation (64) as

$$s_x''' \sim \frac{q_w^K}{T_w^2}\left(1 + \frac{q_w^R}{q_w^K}\right)\left(\frac{T_w - T_\infty}{\delta}\right) \quad (65)$$

or, in terms of the convective heat flux

$$q_x^C = q_w^K \sim k\left(\frac{T_w - T_\infty}{\delta}\right) \quad (66)$$

as

$$s_x''' \sim \frac{k}{T_w^2}\left(1 + \frac{q_w^R}{q_w^K}\right)\left(\frac{T_w - T_\infty}{\delta}\right)^2. \quad (67)$$

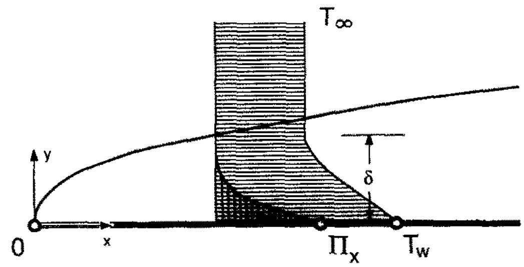

FIG. 4. Wall entropy production.

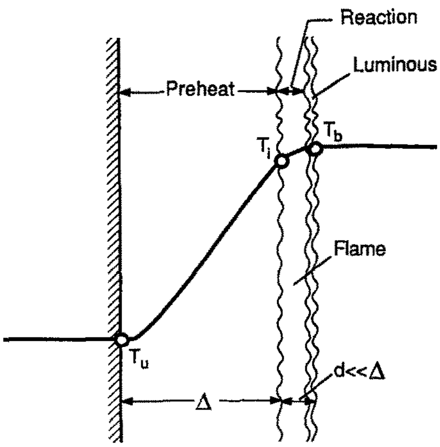

FIG. 5. Quenched laminar flame.

Furthermore, with the definition of local Nusselt number

$$Nu_x = \frac{q_x^C}{q_x^K} = \frac{q_w^K}{q_x^K} \sim \frac{k(T_w - T_\infty)/\delta}{k(T_w - t_\infty)/x} \quad (68)$$

equation (67) may be rearranged as

$$\frac{s_x'''x^2}{k} \sim \left(1 - \frac{T_\infty}{T_w}\right)^2 \left(1 + \frac{q_w^R}{q_w^K}\right) Nu_x^2. \quad (69)$$

Introducing a local entropy production number

$$\Pi_x = s_x'''x^2/k \quad (70)$$

and, for hot wall and small $T_w - T_\infty$, noting $T_\infty/T_w \ll 1$, equation (67) may be further rearranged as

$$\Pi_x \sim \left(1 + \frac{q_w^R}{q_w^K}\right) Nu_x^2 \quad (71)$$

or, in terms of equations (58), (59) and (66), as

$$\Pi_x \sim \left[1 + 4\eta\left(\frac{\varepsilon_w}{2}\right)\left(\frac{\tau}{1+3\tau^2}\right) P_w\right] Nu_x^2. \quad (72)$$

Clearly, the wall entropy production being proportional to Nu_x^2 provides no new information. However, if Nu_x was to be obtained from some approximate temperature profiles, the principle of the 'least entropy production' provides an Nu_x closest to the actual Nu_x. These considerations are related to the well-known theory of variational calculus which is not the concern of this study. The next section deals with another example illustrating the application of entropy production to flames.

7. FLAME QUENCHING

Consider the entropy production in a steady flame stabilized on a porous flat flame burner as depicted in Fig. 5. On dimensional grounds, in a manner similar to equation (64)

$$s''' \sim \frac{1}{T}\left(\frac{q^K + q^R}{T}\right)\left(\frac{T_b - T_u}{\Delta}\right) \quad (73)$$

Δ being the quench distance (the thickness of the reaction zone is d, and $d \ll \Delta$). Rearrange equation (73) as

$$s''' \sim \frac{q^K}{T^2}\left(1 + \frac{q^R}{q^K}\right)\left(\frac{T_b - T_u}{\Delta}\right) \quad (74)$$

or, in terms of

$$q^K \sim k\frac{T_b - T_u}{\Delta} \quad (75)$$

as

$$s''' \sim \frac{k}{T^2}\left(1 + \frac{q^R}{q^K}\right)\left(\frac{T_b - T_u}{\Delta}\right)^2. \quad (76)$$

In view of the fact that most of the reaction occurs close to the highest temperature, use T_b for the characteristic temperature in equation (76). Accordingly

$$s''' \sim \left(1 - \frac{T_u}{T_b}\right)^2 \left(1 + \frac{q^R}{q^K}\right)\frac{k}{\Delta^2} \quad (77)$$

or, in terms of a characteristic length $l = \alpha/S_u^0$, α being the thermal diffusivity and S_u^0 the laminar flame speed at the adiabatic flame temperature, and assume $T_u/T_b \ll 1$

$$\frac{s'''l^2}{k} \sim \left(1 + \frac{q^R}{q^K}\right) Pe^{-2} \quad (78)$$

where

$$Pe = \frac{\Delta}{l} = \frac{S_u^0\Delta}{\alpha} \quad (79)$$

is the flame Peclet number. Separately, when based on the characteristic length $l = \alpha/S_u^0$, the Planck number given by equation (57) yields (in terms of temperatures T_b and T_u)

$$P_b = \frac{E_{bb} - E_{bu}}{k(T_b - T_u)/(\alpha/S_u^0)} \quad (80)$$

which, after some arrangement, becomes the flame Boltzmann number

$$B_b = \frac{E_{bb} - E_{bu}}{\rho c_p S_u^0 (T_b - T_u)} \sim \frac{\text{Emission}}{\text{Flame enthalpy flow}}. \quad (81)$$

Thus, in view of the relation

$$\frac{P_b}{q^K(Pe)} \equiv B_b \quad (82)$$

equations (60), (79) and (81) lead to

$$\Pi_s \sim \frac{1}{Pe^2} + 4\eta\left(\frac{\varepsilon_w}{2}\right)\left(\frac{\tau}{1+3\tau^2}\right)\frac{B_b}{Pe}. \quad (83)$$

The linearized B_b is independent of the flame temperature, or, with the approximation

$$\frac{E_b - E_u}{T_b - T_u} \sim \frac{E_b^0 - E_u}{T_b^0 - T_u}$$

B_b itself becomes independent of this temperature. Thus

$$\Pi_s = f(\eta\varepsilon_w, \tau, B_b, Pe) \tag{84}$$

where

$$Pe = f(\Delta) \quad \text{and} \quad \Delta = f(T_b)$$

and Π_s depends on the flame temperature only through the Peclet number (dimensionless quench distance).

The U-shaped nature of $\Delta = f(T_b)$ is well documented in the literature (see Ferguson and Keck [23, 24] for the case of excluding radiation and Arpaci and Tabaczynski [14, 15] for the case with radiation; also, see Kooker [25] and Sohrab and Law [26] for the importance of radiation on the quenching process, and Lee and Tien [27] for the effect of condensed fuels on this process). References [14, 15, 23, 24] follow the usual practice and evaluate the minimum quench distance from the *tangency condition*

$$\frac{\partial}{\partial T_b}(Pe) = 0. \tag{85}$$

Actually, *an extremum of the entropy production*

$$\frac{\partial \Pi_s}{\partial T_b} \sim -\left[\frac{2}{Pe^3} + 4\eta\left(\frac{\varepsilon_w}{2}\right)\left(\frac{\tau}{1+3\tau^2}\right)\frac{B_b}{Pe^2}\right]\frac{\partial}{\partial T_b}(Pe) = 0 \tag{86}$$

provides the physical justification for this condition (note that the terms in brackets are positive).

8. CONCLUSIONS

The concept of lost heat is originated as opposed to that of lost work. It is shown that all forms of energy are dissipated into heat and describe the non-thermal part of entropy production while the heat energy is dissipated into entropy and describes the thermal part of this production. A dimensionless number for entropy production is introduced. This number is evaluated in terms of two illustrative cases. The first case involves the entropy production on the wall of a thermal boundary layer. This production is found to be proportional to the square of the Nusselt number. Unless it is tied to a variational problem which selects the physically meaningful solution among all mathematically possible solutions, the entropy production provides no new information for this case. The second case involves the entropy production in the luminous zone of a quenched flame. The production is found to be *inversely* proportional to the Peclet number. The *tangency condition*, usually considered in the literature to determine the minimum quench distance, is related to *an extremum in entropy production*.

Although the entropy production in radiating gases continues to remain untreated, it is worth mentioning the considerable size of the literature on entropy production in enclosure radiation (non-participating media) and its solar application. For early works, refer to Spanner [28] and Petela [29]. For the latest studies, see Gribik and Osterle [30] and the references cited therein.

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PRODUCTION D'ENTROPIE PAR RAYONNEMENT—PERTE DE CHALEUR ET ENTROPIE

Résumé—Le flux thermique δQ du premier principe de la thermodynamique est exprimé en fonction du flux d'entropie $\delta(Q/T)$:

$$\delta Q \equiv \delta[T(Q/T)] = T\delta(Q/T)+(Q/T)\,\mathrm{d}T$$

où $T\delta(Q/T)$ est l'équivalent énergétique du flux d'entropie et $(Q/T)\,\mathrm{d}T$ introduit le concept de production d'entropie par flux de chaleur. Ici $Q = Q^K+Q^R$, ou K et R indiquent respectivement conduction et rayonnement. En terme de chaleur perdue, les productions d'entropie adimensionnelles sur la paroi d'une couche limite thermique et dans une flamme laminaire sont respectivement

$$\Pi_x \sim (1+q_x^R/q_x^K)Nu_x^2 \quad \text{et} \quad \Pi_s \sim (1+q^R/q^K)Pe^{-2}$$

où q^R et q^K sont les flux monodimensionnels associés à Q^R et Q^K, Nu_x est un nombre de Nusselt local et Pe est un nombre de Peclet basé sur la vitesse de flamme laminaire à la température adiabatique. La *condition de tangente* $\partial Pe/\partial T_b = 0$, habituellement utilisée sans justification physique dans l'evaluation de la distance est montrée correspondre à un *extrémum de production d'entropie.*

ENTROPIEERZEUGUNG BEI STRAHLUNG AUS "VERLORENER WÄRME"

Zusammenfassung—Der Wärmestrom δQ aus dem 1. Hauptsatz der Thermodynamik wird mit Hilfe des Entropiestromes $\delta(Q/T)$ ausgedrückt:

$$\delta Q \equiv \delta[T(Q/T)] = T\delta(Q/T)+(Q/T)\,\mathrm{d}T$$

wobei $T\delta(Q/T)$ das Energieäquivalent zum Entropiestrom darstellt und $(Q/T)\,\mathrm{d}T$ die Konzeption der "Entropieerzeugung aus verlorener Wärme" einführt. Es gilt $Q = Q^K+Q^R$, wobei die Indizes K und R für Leitung bzw. Strahlung stehen. Mit den Bezeichnungen der "verlorenen Wärme" lassen sich die Entropieproduktion an der Wand unter einer thermischen Grenzschicht und in einer verlöschten laminaren Flamme folgendermaßen schreiben:

$$\Pi_x \sim (1+q_x^R/q_x^K)Nu_x^2 \quad \text{und} \quad \Pi_s \sim (1+q^R/q^K)Pe^{-2}$$

q^R und q^K sind die flächenbezogenen Werte von Q^R und Q^K, Nu_x die örtliche Nusselt-Zahl, Pe eine Peclet-Zahl, gebildet mit der laminaren Flammengeschwindigkeit bei der adiabaten Flammentemperatur. Die Tangentenbedingung, $\partial Pe/\partial T_b = 0$, üblicherweise zur Berechnung von minimaler Kühldistanz ohne jede physikalische Rechtfertigung benutzt, erweist sich als Extremum bei der Entropieerzeugung.

ПРОИЗВОДСТВО ЭНТРОПИИ ИЗЛУЧЕНИЯ—ТЕПЛОПОТЕРИ КАК ПРИРОСТ ЭНТРОПИИ

Аннотация—В соответствии с первым законом термодинамики тепловой поток δQ может быть выражен через поток энтропии $\delta(Q/T)$

$$\delta Q \equiv \delta[T(Q/T)] = T\delta(Q/T) + (Q/T)\,\mathrm{d}T$$

где $T\delta(Q/T)$ обозначает энергию, эквивалентную потоку энтропии, а $(Q/T)\,\mathrm{d}T$ вводит понятие теплопотери как прирост энтропии. Здесь $Q = Q^K + Q^R$, где верхние индексы K и R соответственно обозначают теплопроводность и излучение. Показано, что пользуясь понятием теплопотери, безразмерный прирост энтропии на внешней границе теплового пограничного слоя и в гаснущем ламинарном пламени может быть представлен как

$$\Pi_x \sim (1 + q_x^R/q_x^K)Nu_x^2 \quad \text{и} \quad \Pi_s \sim (1 + q^R/q^K)Pe^{-2}$$

где q^R и q^K—одномерные потоки, связанные с Q^R и Q^K; Nu_x—локальное число Нуссельта, а Pe—число Пекле для скорости ламинарного пламени при адиабатической температуре пламени. Показано также, что *условие экстремума* $\partial Pe/\partial T_b = 0$, обычно используемое при определении расстояния, на котором происходит гашение пламени, без какого-либо физического обоснования соответствует *экстремальному значению прироста энтропии.*